A diffusive wander through human life

Moshe Milevsky takes a random walk through the human life cycle and reflects on how quantitative finance is transforming the field of personal wealth management.

Introduction

When I mention the terms *Ito's lemma* or *diffusion processes* to our new graduate students in finance and economics—upon being asked about the tools of my research trade—they immediately assume that I work in the esoteric field of option pricing and derivative hedging. Most are surprised to learn that I actually use these concepts to analyse practical questions such as how much and which type of life insurance people should purchase, the best age at which to start drawing a retirement pension annuity or the optimal home mortgage loan for their family. Indeed, the models of quantitative finance that spawned a derivatives revolution in the 1970s and 1980s, and then filtered through to real options in corporate finance and strategy in the 1980s and 1990s, have finally arrived on the layman's doorstep. The objective is crystal clear: namely, to help individuals make better personal wealth and risk management decisions.

In the last decade financial luminaries such as Bill Sharpe, Harry Markowitz and Robert Merton have been penning articles in the nascent field of what I like to call quantitative wealth management (QWeMa). This article is a brief survey of this emergent field. Please refer to the bibliography at the end for a sample of some key pieces in this growing and exciting field.

A number of factors have been driving the 'quantification' of personal wealth and risk management, but the primary culprit has been the spiral of wealth management products and choices that can leave even the most seasoned financial advisor dazed and confused. Add this growing menu to the fiduciary responsibility faced by many advisors who do not want to (appear to) make arbitrary and possibly biased financial decisions, and the need for expert tools and systems becomes evident.

So let us take a random walk through the human life cycle of QWeMa.

Portfolio selection with human capital

While the classical Markowitz theory and ubiquitous efficient frontier predates much of modern financial economics, the application to human capital and broader wealth only began recently. To understand human capital and its role in portfolio selection, we must take a step back and imagine the individual investor/consumer—the *raison d'être* of QWeMa—as a small, privately held and poorly diversified corporation called 'You Incorporated'. Table 1 provides a snapshot of the balance sheet.

Note that, in contrast to Financial Accounting Standards Board (FASB) or Generally Accepted Accounting Principles (GAAP) standards applied to publicly traded companies, we place an asset called human capital (HC) on the left-hand side of the balance sheet. HC is a measure of the present value of future wages, income and salary that will be earned over the human life cycle. You cannot touch, feel or see HC but, like an oil reserve deep under the ocean, it will eventually be extracted, so it is worth something today. More importantly, it has its own risk and return characteristics and can be modelled as a random variable that is correlated with a variety of economic factors. You can think of your HC as a continuous-time stochastic process which is subject to unpredictable jumps and discontinuities. This process pays a lumpy stream of dividends, eventually converges to zero—if you are still alive—and your objective is to maximize these dividends in some risk-adjusted manner. Remember that you can control your HC process by investing in education, making prudent career choices (and healthy eating).

Can a course called 'Ito Calculus for Lawyers' be far behind?

Now, with this preliminary framework in mind, recall the Markowitz paradigm which dictates that total capital—both human and financial—should be properly diversified. Thus, in the early stages of the life cycle, financial capital and investments should be used to hedge and diversify HC. Think of investable assets as a defence against adverse salaries and wages as opposed to an isolated pot of money that has to be allocated.

To understand the implications of modelling HC as a random variable, let us take an example. For a tenured university professor like myself, HC—and the subsequent defined benefit pension I am entitled to—has the risk and return properties of a fixed-income bond fund paying monthly coupons. In a sense, I am a walking inflation-adjusted real return bond. So, I have very little need for fixed-income bonds, money market funds and other guaranteed products in my discretionary financial portfolio. In fact, my personal savings are heavily invested in individual equities and broad-based investment funds. The opposite advice would be relevant for the MBA and graduate students I teach, who intend to work in the financial services sector

Table 1. The balance sheet of *You Inc*.

Assets	Debts and liabilities
Bank accounts	Mortgages
Housing	Consumer loans
Stocks and bonds	Credit cards
Car and vehicles	Student loans
Small business equity	
Potential value of pension	
+ Human capital	Total personal equity = True net worth

when they graduate. Thus, one of the fertile areas of research in QWeMA revolves around constructing a financial portfolio—where to invest your personal savings—so that your total portfolio is properly diversified. This line of research examines the wage profile of various occupations and then models their covariation with general financial markets to arrive at tailor-made investment solutions based on unique idiosyncratic factors.

Yet, QWeMa goes far beyond tailoring personalized investment portfolios for dentists, lawyers and doctors. The ideas can be used to examine and think about a number of other issues.

Asset location versus asset allocation

Although income tax regulations are country specific, most jurisdictions offer individual tax shelters and concessions for long-term savings for which capital gains, dividends and even interest income is tax-deferred. These products may also be associated with personal pension plans or be part of a stand-alone life insurance and savings policy. Regardless of the exact form, these strategies often leave individual investors with a difficult decision regarding which assets should be subjected to which tax treatment. This is known as the asset location problem, as opposed to the asset allocation problem, which has spawned yet another interesting avenue of research within QWeMa. From a mathematical point of view, these multi-period optimization problems are highly path-dependent due to peculiar tax asymmetries and they quickly escalate in dimensionality. For example, while the gains on a stock are usually taxed more favourably than gains on a bond, if they are placed within tax shelters and held for short periods the situation may be reversed if, upon exiting the shelter, the income is penalized in some manner. Indeed, personal income taxes can severely distort optimal trading, hedging and investment strategies far beyond the perfect and frictionless markets envisioned by the purists.

Home mortgage financing

Beyond investment and portfolio issues, most individuals purchase a personal residence at some point in their life, yet very few have the liquid wealth to pay for this transaction in cash. Mortgage financing is the way in which this leveraged positioned is usually accomplished. And, just as publicly traded companies face a myriad of choices between fixed and floating rate obligations in their capital structure, individuals must decide on the composition and characteristics of their debt. Depending on the jurisdiction and country, consumers must decide whether to go 'long' or 'short' term or whether to 'close' or leave 'open' their mortgage. In some cases, individuals must further decide whether to borrow in real (after-inflation) or nominal terms or whether to link their payments to a given index. In all of these personal deliberations, a robust and dynamic model for the evolution of the term-structure of interest rate is required. However, the models must do more than explain the evolution of the curve over short periods of time and under the Q-measure, akin to what an option trader might need. They must also provide realistic long-term P-measure forecasts and risk metrics taking into account the option to default, prepay and move. The presence of complex penalties and prepayment charges further fuels the demand for rigorous and sound advice.

Life and health insurance

Personal risk management is an exercise in protecting human as well as financial capital: both forms of capital should diversify each other. So property insurance should also be viewed as a hedge against the loss of financial capital and life insurance should be treated as a hedge against the unsupportable loss—to the surviving family—of HC. Thus, all forms of life, health and disability insurance should not be acquired as an investment, rather they should be purchased for their hedging properties.

[The] age at which to get married, have children and get divorced all become an exercise in American option pricing

If something happens to HC, the insurance will payoff, but if nothing (bad) happens to HC, the insurance will perform poorly. Insurance is an asset class with a negative expected rate of return (on a pre-tax basis, at least) but nevertheless forms part of the optimal portfolio because of the negative correlation matrix. The concept might sound simple but the broad menu of insurance choices available—such as term life, whole life and universal life—generates yet another difficult portfolio selection problem. In many cases, insurance products provide guaranteed renewability and contain implicit options on mortality tables. Adverse selection, which is at the heart of insurance pricing, becomes an optimal strategy in its own right.

Employee compensation and proper incentives

QWeMa also gives insights into the optimal design of employment compensation contracts. Trends towards company stock-linked compensation and incentive stock options create a need for normative advice on optimal exercise strategies in the presence of highly illiquid and concentrated portfolios. Furthermore, if giving more options to a greater number of employees only serves to increase their risk profile, the incentive effects may be counterproductive. A sound mathematical model of the tradeoff between the various risk factors is required for careful analysis. Indeed, the recent accounting debate surrounding the cost of an incentive stock option must be based on a solid understanding of how individuals value these illiquid instruments.

Choice of defined benefit versus defined contribution pensions

For years, the venerable defined benefit (DB) pension plan was the mainstay of retirement pension provision. An individual worked for the same company during his/her entire life, and earned a pension annuity based on career average earnings and/or the number of years of credited service. In DB plans the 'risk' was borne by the employer who had to provide and support the guarantee. Over the last 10–15 years there has been a sharp and noticeable shift towards defined contribution (DC) pension plans. Under these plans, employees are entitled to fixed periodic contribution (with full control over its allocation) but the risk is squarely in their hands. Although it is debatable what is best for individuals at which point in their life, the tran-

sition process from DB to DC involves a number of choices amenable to mathematical modelling. For example, what is the value of the option to choose which plan to belong to? Or what is the optimal mix between DB and DC plans from a portfolio perspective? Actuaries have only recently started to embrace the preachings of financial economics for the valuation of pension benefits and QWeMa models are intrinsic to the analysis.

Starting a retirement pension annuity

The decision of when to retire can lead to some fascinating and complicated optimal stopping problems. On one hand we have the illiquidity and irreversibility of a pension annuity which leads people to delay annuitization. On the other hand, we are faced with lifetime uncertainty and the desire to hedge against longevity risk. A number of recent models have been developed to help consumers understand and appreciate these tradeoffs. Once again, we must turn to a holistic risk and wealth management framework to provide proper guidance.

Estate planning and tax arbitrage

Many countries impose estate and wealth taxes on individuals upon their demise. And while death and taxes are truly unavoidable, the ability to minimize the impact of the former on the latter is a rich and fertile endeavour for many lawyers and accountants specializing in estate transfers. Interestingly, a number of provisions within the tax code allow for mortality and tax arbitrage by allowing portions of the estate to escape taxation. The precise construction and implementation of these strategies often comes down to satisfying certain internal rate of return criteria or probabilistic thresholds that, once again, fall naturally within the domain of QWeMa. (Can a course called 'Ito Calculus for Lawyers' be far behind?)

The real options in your life

From a broader perspective, many of the above-mentioned optimization problems involve an element of timing and irreversibility, which bring to mind the classical call and put option analogy. An American option differs from its European counterpart in that one can choose when to exercise the option. Taking this concept further, we can frame many decisions, financial as well as non-financial, that people face, as option pricing problems. Viewed from this perspective, the optimal career, time to purchase a house and even age at which to get married, have children and get divorced all become an exercise in American option pricing, or at the very least a dynamic programming problem. Time constraints prevent an in-depth elaboration, but one can imagine the numerous additional applications.

One of the defining features of QWeMa is that, in contrast to recent work in behavioural financial economics which sheds light on the psychological biases that lead to human mistakes, we are taking a normative perspective and trying to help people make better decisions. To summarize, the field of quantitative wealth management has its best years ahead, both in terms of academic developments and application by industry.

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Moshe Arye Milevsky is at the Schulich School of Business at York University and the Executive Director of the Individual Finance and Insurance Decisions Centre at the Fields Institute for Research in Mathematical Science, Toronto, Canada.